

DAK-91-22

OBJECTIVE

NOTE: You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

QUESTION NO. 1

- 1 $\lim_{x \rightarrow +\infty} \frac{2-3x}{\sqrt{3+4x^2}} = \dots\dots\dots$
(A) $3/2$ (B) $-3/2$ (C) $+\infty$ (D) $-\infty$
- 2 If $f(x) = \begin{cases} x+2 & x \leq -1 \\ c+2 & x > -1 \end{cases}$ and $\lim_{x \rightarrow -1} f(x)$ exists then $c = \dots\dots\dots$
(A) -2 (B) 2 (C) 1 (D) -1
- 3 $\frac{d}{dx} \sin^{-1} x = \dots\dots\dots$
(A) $\frac{1}{\sqrt{1+x^2}}$ (B) $\frac{-1}{\sqrt{1+x^2}}$ (C) $\frac{1}{\sqrt{1-x^2}}$ (D) $\frac{-1}{\sqrt{1-x^2}}$
- 4 Any point where function f is neither increasing nor decreasing provided $f'(x) = 0$ is called
(A) Critical point (B) Point of inflection (C) Stationary point (D) Feasible point
- 5 $\frac{d}{dx} \cos(ax+b) = \dots\dots\dots$
(A) $\sin(ax+b)$ (B) $-a \sin(ax+b)$ (C) $a \sin(ax+b)$ (D) $-\sin(ax+b)$
- 6 $\frac{d}{dx} e^{3x} = \dots\dots\dots$
(A) $\frac{1}{3} e^{3x}$ (B) e^{3x} (C) $3 e^{3x}$ (D) $3 e^{3x} \ln 3$
- 7 $\int_{-\pi}^{\pi} \sin x \, dx = \dots\dots\dots$
(A) -1 (B) 0 (C) 1 (D) $\cos x$
- 8 $\int \frac{e^x}{e^x+3} \, dx = \dots\dots\dots$
(A) $e^x + 3 + c$ (B) $e^x + c$ (C) $e^x \ln(e^x+3) + c$ (D) $\ln(e^x+3) + c$
- 9 $\int \frac{x}{\sqrt{4+x^2}} \, dx = \dots\dots\dots$
(A) $\sqrt{4+x^2} + c$ (B) $\frac{1}{2} \sqrt{4+x^2}$ (C) $\frac{1}{(x+4)^{3/2}} + c$ (D) $\ln|\sqrt{4+x^2}| + c$
- 10 Solution of differential equation $\frac{dy}{dx} = -y$ is
(A) $y = -ce^x$ (B) $y = ce^x$ (C) $y = ce^{-x}$ (D) $y = e^x$
- 11 The distance between the points A (3, 1), B (-2, -4)
(A) $2\sqrt{5}$ (B) $5\sqrt{2}$ (C) $\sqrt{5}$ (D) $\sqrt{2}$
- 12 The point of intersection of the lines $3x + y + 12 = 0$ and $x + 2y - 4 = 0$ is
(A) (5, 3) (B) (-5, -3) (C) (5, -3) (D) (-5, 3)
- 13 Slope of the line $2x + 5y - 8 = 0$ is
(A) $-2/5$ (B) $2/5$ (C) $5/2$ (D) $-5/2$
- 14 The y-intercept of the equation of line $5x - 12y + 39 = 0$
(A) $\frac{5}{12}$ (B) $\frac{-39}{12}$ (C) $\frac{39}{12}$ (D) $\frac{-5}{12}$
- 15 Graph of the inequality $x + 2y < 6$ lies $\dots\dots\dots$
(A) Opposite to origin (B) Toward origin (C) in 1st quadrant (D) in 2nd quadrant
- 16 Radius of the circle with equation $x^2 + y^2 + 2gx + 2fy + c = 0$ is
(A) $\sqrt{g^2 + f^2 + c}$ (B) $\sqrt{g^2 - f^2 - c}$ (C) $\sqrt{g^2 + f^2 - c^2}$ (D) $\sqrt{g^2 + f^2 - c}$
- 17 The line through the focus and perpendicular to the directrix of parabola is called
(A) tangent to parabola (B) axis of parabola (C) latusrectum of parabola (D) vertex of parabola
- 18 $x = a \cos \theta$, $y = b \sin \theta$ are parametric equations of $\dots\dots\dots$
(A) Circle (B) Parabola (C) Ellipse (D) Hyperbola
- 19 If \underline{u} and \underline{v} be two vectors making an angle θ with each other then projection of \underline{u} along \underline{v} is
(A) $\frac{\underline{u} \cdot \underline{v}}{|\underline{v}|}$ (B) $\frac{\underline{u} \cdot \underline{v}}{|\underline{u}|}$ (C) $\frac{\underline{u} \times \underline{v}}{|\underline{v}|}$ (D) $\frac{\underline{u} \times \underline{v}}{|\underline{u}|}$
- 20 $3\hat{j} \cdot \hat{k} \times \hat{i} = \dots\dots\dots$
(A) 0 (B) -3 (C) \hat{j} (D) 3

QUESTION NO. 2 Write short answers any Eight (8) of the following D.K.-91-22 16

| | |
|------|--|
| i | Prove the identity $\sec^2 x = 1 + \tan^2 x$ |
| ii | If $f(x) = 2x + 1$ and $g(x) = x^2 - 1$. Then obtain the expression $fg(x)$ |
| iii | Obtain $f^{-1}(x)$ from $f(x) = -2x + 8$ |
| iv | Evaluate $\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x}$ |
| v | If $f(x) = \begin{cases} x + 2, & x \leq -1 \\ c + 2, & x > -1 \end{cases}$, find "c" so that $\lim_{x \rightarrow -1} f(x)$ exists |
| vi | If $y = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$, then find $\frac{dy}{dx}$ |
| vii | Differentiate $x^2 - \frac{1}{x^2}$ w.r.t. " x^4 " |
| viii | If $y = x^2 \sec 4x$, then find $\frac{dy}{dx}$ |
| ix | Obtain $\hat{f}(x)$ from $f(x) = x^3 \cdot e^{1/x}$ |
| x | Find $\frac{dy}{dx}$ if $y = x e^{\sin x}$ |
| xi | Determine the interval in which $f(x) = 4 - x^2$, $x \in (-2, 2)$ is increasing |
| xii | Examine the function $f(x) = x^2 - x - 2$ for critical values |

QUESTION NO. 3 Write short answers any Eight (8) of the following 16

| | |
|------|--|
| i | Use differentials to find $\frac{dy}{dx}$ if $xy + x = 4$ |
| ii | Find $\int \frac{dx}{\sqrt{x+1} - \sqrt{x}}$ |
| iii | Find $\int x \cdot \sqrt{x^2 - 1} dx$ |
| iv | Find $\int \frac{x^2}{x^2+4} dx$ |
| v | Find $\int \tan^{-1} x dx$ |
| vi | Find $\int e^{-x} (\cos x - \sin x) dx$ |
| vii | $\int_{-6}^2 \sqrt{3-x} dx$ |
| viii | Solve the differential equation $\frac{dy}{dx} = \frac{y}{x^2}$ |
| ix | Find the equation of a vertical line through $(-5, 3)$ |
| x | Convert the equation $2x - 4y + 11 = 0$ (i) Two intercepts form (ii) Normal form |
| xi | Check whether the point $(5, 8)$ lies below or above the line $2x - 3y + 6 = 0$ |
| xii | Find the lines represented by $3x^2 + 7xy + 2y^2 = 0$ |

QUESTION NO. 4 Write short answers any Nine (9) of the following 18

| | |
|------|---|
| i | Graph the solution set of $3x - 2y \geq 6$ |
| ii | Graph the solution set of the following linear inequality $3x + 7y \geq 21, y \leq 4$ |
| iii | If $\underline{v} = \frac{-\sqrt{3}}{2} \underline{i} - \frac{1}{2} \underline{j}$, then find a unit vector in the direction of \underline{v} |
| iv | If $\underline{u} = 2\underline{i} + 3\underline{j} + \underline{k}$, $\underline{v} = 4\underline{i} + 6\underline{j} + 2\underline{k}$ and $\underline{w} = -6\underline{i} - 9\underline{j} - 3\underline{k}$ then find $\underline{u} + 2\underline{v}$ |
| v | If $\underline{a} = 2\underline{i} - 2\underline{j} + 4\underline{k}$, $\underline{b} = -\underline{i} + \underline{j} - 2\underline{k}$ then find a unit vector perpendicular to plane containing \underline{a} and \underline{b} |
| vi | If $\underline{u} = \underline{i} + 2\underline{j} - \underline{k}$, $\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$ and $\underline{w} = \underline{i} - 7\underline{j} - 4\underline{k}$. Then find volume of parallelepiped by these vectors |
| vii | Find work done, if the point at which the constant force $\underline{F} = 4\underline{i} + 3\underline{j} + 5\underline{k}$ is applied to an object moves from $P_1(3, 1, -2)$ to $P_2(2, 4, 6)$ |
| viii | Write equation of normal to the circle $x^2 + y^2 = 25$ at $(5 \cos \theta, 5 \sin \theta)$ |
| ix | Find focus of the parabola $x^2 - 4x - 8y + 4 = 0$ |
| x | Find eccentricity and vertices of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ |
| xi | Define circle and write equation of circle in standard form |
| xii | Find equation of the parabola with focus $(2, 5)$ and directrix $y = 1$ |
| xiii | Find centre and foci of the hyperbola $\frac{y^2}{4} - x^2 = 1$ |

SECTION-II

10 x 3 = 30

Note: Attempt any Three questions from this section.

Dzk-41-22

| | |
|----------|---|
| Q.5- (A) | If $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$, then show $y \frac{dy}{dx} + x = 0$ |
| (B) | Find m and n so that the given function is continuous at $x = 3$ if $f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$ |
| Q.6- (A) | Find $\int \sin^4 x \, dx$ |
| (B) | Find the equation of the line through $(5, -8)$ and perpendicular to join of $A(-15, -8)$, $B(10, 7)$ |
| Q.7-(A) | Evaluate $\int_{-1}^2 (x + x) \, dx$ |
| (B) | Maximize $f(x,y) = x + 3y$; subject to the constraints $2x + 5y \leq 30$, $5x + 4y \leq 20$, $x \geq 0$, $y \geq 0$ |
| Q.8-(A) | Find the area of the region bounded by the triangle whose sides are $7x - y - 10 = 0$; $10x + y - 41 = 0$; $3x + 2y + 3 = 0$ |
| (B) | Determine the equations of tangents to the circle $x^2 + y^2 = 2$ perpendicular to the line $3x + 2y = 6$ |
| Q.9-(A) | By transforming the equation $x^4 + 4y^2 - 2x + 8y + 4 = 0$ referred to a new origin and axes remaining parallel to the original axes, the first terms are removed. Find the coordinates of the new origin and the transformed equation |
| (B) | Prove that: $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$ |

NOTE: You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

QUESTION NO. 1

D9K 92-22

1. $\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^n = \dots\dots\dots$
 (A) e^n (B) e (C) $e^{1/2}$ (D) $e^{-1/2}$
2. If $f(x) = 2x + 1$, $g(x) = \frac{3}{x-1}$, $x \neq 1$ then $f \circ g(x) = \dots\dots\dots$
 (A) $\frac{5+x}{x-1}$ (B) $\frac{3}{2x}$ (C) $4x + 3$ (D) $\frac{3(x-1)}{4-x}$
3. $\frac{d}{dx} \cot h^{-1} x = \dots\dots\dots$
 (A) $\frac{1}{1-x^2}$ (B) $\frac{1}{x^2-1}$ (C) $\frac{1}{1+x^2}$ (D) $\frac{-1}{1+x^2}$
4. $\frac{d}{dx} (x+4)^{1/3} = \dots\dots\dots$
 (A) $(x+4)^{-1/3}$ (B) $\frac{1}{3}(x+4)^{-1/3}$ (C) $\frac{1}{3}(x+4)^{-2/3}$ (D) $\frac{1}{3}(x+4)^{2/3}$
5. $\frac{d}{dx} e^{\sin x} = \dots\dots\dots$
 (A) $e^{\sin x}$ (B) $\cos x e^{\sin x}$ (C) $\sin x e^{\sin x-1}$ (D) $-\cos x e^{\sin x}$
6. If f be a differentiable function on the open interval (a, b) then f is increasing function if
 (A) $f'(x) < 0$ (B) $f'(x) > 0$ (C) $f(x) \leq 0$ (D) $f''(x) < 0$
7. $\int \frac{1}{ax+b} dx = \dots\dots\dots$
 (A) $\ln |ax+b| + c$ (B) $\frac{ax+b}{a} + c$ (C) $\frac{-a}{(ax+b)^2} + c$ (D) $\frac{1}{a} \ln |ax+b| + c$
8. $\int (f(x))^{-1} f'(x) dx = \dots\dots\dots$
 (A) $\ln |f(x)| + c$ (B) $\frac{[(f(x))^{-1}]^2}{2} + c$ (C) $(f(x))^{-1} + c$ (D) $f(x) + c$
9. $\int \tan^2 x dx = \dots\dots\dots$
 (A) $\sec^2 x + c$ (B) $\sec^2 x - x + c$ (C) $x - \sec^2 x + c$ (D) $-\operatorname{cosec}^2 x + c$
10. Solution of the differential equation $x \frac{dy}{dx} = 1 + y$ is
 (A) $c - \frac{1}{x}$ (B) ce^y (C) $y = cx - 1$ (D) $x^2 + y^2 = c$
11. Equation of horizontal line through $(7, -9)$ is
 (A) $y = -9$ (B) $y = 9$ (C) $x = 7$ (D) $x = -7$
12. Slope intercept form of the line $2x + y - 11 = 0$ is
 (A) $\frac{x}{(11/2)} + \frac{y}{11} = 1$ (B) $y = -2x + 11$ (C) $y = 2x - 11$ (D) $y = -2x - 11$
13. If $\theta = 45^\circ$ be the inclination of the line with x -axis then slope of the line is
 (A) $\frac{-1}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$ (C) -1 (D) 1
14. The equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of orthogonal lines if
 (A) $h^2 - ab = 0$ (B) $a + b = 0$ (C) $h^2 + ab = 0$ (D) $a - b = 0$
15. The non-negative constraints used in a system of linear inequalities are called
 (A) Problem constraints (B) Decision variable (C) Feasible solution (D) Optimal solution
16. Co-ordinate of the centre of the circle $x^2 + y^2 + 12x - 10y = 0$ is
 (A) $(6, -5)$ (B) $(-6, -5)$ (C) $(-6, 5)$ (D) $(6, 5)$
17. Focus of the parabola $x^2 = -4ax$ is
 (A) $(0, -a)$ (B) $(0, a)$ (C) $(-a, 0)$ (D) $(a, 0)$
18. Equation of Directrices of Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
 (A) $y = 0$ (B) $x = 0$ (C) $y = \pm \frac{c}{e^2}$ (D) $x = \pm \frac{c}{e^2}$
19. The value of $[\underline{i} \ \underline{j} \ \underline{k}] = \dots\dots\dots$
 (A) 1 (B) 0 (C) -1 (D) \underline{k}
20. With usual notations in any triangle ABC $c \cos A + a \cos C = \dots\dots\dots$
 (A) a (B) b (C) c (D) 1

QUESTION NO. 2 Write short answers any Eight (8) of the following **D9K-92-22** 16

| | |
|------|---|
| i | If $f(x) = x^2 - x$, Evaluate $f(x-1)$ |
| ii | Explain Identity function by example |
| iii | Evaluate $\lim_{x \rightarrow \pi} \frac{\sin x}{\pi - x}$ |
| iv | Show that $x = a \cos t$ and $y = a \sin t$ are the parametric equation of the circle $x^2 + y^2 = a^2$ |
| v | Express $\lim_{n \rightarrow +\infty} \left(1 + \frac{3}{n}\right)^{2n}$ in terms of e |
| vi | If $x = t^2 + 1$, $y = t^2$ find $\frac{dy}{dx}$ |
| vii | If $3x + 4y + 7 = 0$ then find $\frac{dy}{dx}$ |
| viii | Differentiate $\frac{1}{a} \sin^{-1} \frac{a}{x}$ w.r.t x |
| ix | Find y_2 if $x^2 + y^2 = a^2$ |
| x | Explain increasing function and give its example |
| xi | Differentiate $\sin x$ w.r.t $\cot x$ |
| xii | Calculate $\frac{d}{dx} (3x^{4/3})$ |

QUESTION NO. 3 Write short answers any Eight (8) of the following 16

| | |
|------|--|
| i | Evaluate $\int \frac{\cos 2x - 1}{1 + \cos 2x} dx$ |
| ii | Evaluate $\int a^{x^2} x dx$ |
| iii | Evaluate $\int \frac{dx}{\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x}$ |
| iv | Evaluate $\int (e^n x)^2 dx$ |
| v | Find $\int_{-1}^3 (x^3 + 3x^2) dx$ |
| vi | If $\int_{-2}^1 f(x) dx = 5$ and $\int_{-2}^1 g(x) dx = 4$ Then evaluate $\int_{-2}^1 (2f(x) + 3g(x)) dx$ |
| vii | Find area between the x-axis and the curve $y = 4x - x^2$ |
| viii | Check $y = \tan(e^x + c)$ is a solution of the differential equation of $\frac{dy}{dx} = \frac{y^2 + 1}{e^{-x}}$ |
| ix | If the vertices of a triangular region are $A(5, 3)$, $B(-2, 2)$ and $C(4, 2)$. Find its area |
| x | Convert $5x - 12y + 39 = 0$ into slope intercept and intercept form |
| xi | Find the point three-fifth of the way along line segment from $A(-5, 8)$ to $B(5, 3)$ |
| xii | By means of slope show that the points $(4, -5)$, $(5, 7)$ and $(10, 15)$ lies on a same line |

QUESTION NO. 4 Write short answers any Nine (9) of the following 18

| | |
|------|---|
| i | Graph the solution set of linear inequality $3y - 4 \leq 0$ in xy - plane |
| ii | Define feasible region and feasible solution |
| iii | Find an equation of the circle with centre at $(\sqrt{2}, -3\sqrt{3})$ and radius $2\sqrt{2}$ |
| iv | Find the focus and directrix of the parabola $y^2 = -8(x - 3)$ |
| v | Find an equation of the ellipse with foci $(-3\sqrt{3}, 0)$ and vertices $(\pm 6, 0)$ |
| vi | Find focus of the parabola $x^2 - 4x - 8y + 4 = 0$ |
| vii | Check the position of the point $(5, 6)$ with respect to the circle $x^2 + y^2 = 81$ |
| viii | Find the centre and radius of the circle $4x^2 + 4y^2 - 8x + 12y - 25 = 0$ |
| ix | Write the vector \underline{PQ} in the form $x\underline{i} + y\underline{j}$ if $P(0, 5)$, $Q(-1, -6)$ |
| x | Find the sum of the vectors \underline{AB} and \underline{CD} given that four points $A(1, -1)$, $B(2, 0)$, $C(-1, 3)$, $D(-2, 2)$ |
| xi | Find a unit vector in the direction of $\underline{V} = \underline{i} + 2\underline{j} - \underline{k}$ |
| xii | Find the cosines of angle θ between $\underline{U} = [2, -3, 1]$, $\underline{V} = [2, 4, 1]$ |
| xiii | Prove that $\underline{a} \times (\underline{b} + \underline{c}) + \underline{b} \times (\underline{c} + \underline{a}) + \underline{c} \times (\underline{a} + \underline{b}) = 0$ |

SECTION-II

Note: Attempt any Three questions from this section

10 x 3 = 30

09K-92-22

| | |
|----------|--|
| Q.5- (A) | Find $\lim_{\theta \rightarrow 0} \frac{\tan\theta - \sin\theta}{\sin^3\theta}$ |
| (B) | Prove that if $\frac{y}{x} = \tan^{-1} \frac{x}{y}$ then $\frac{dy}{dx} = \frac{y}{x}$ |
| Q.6- (A) | Evaluate $\int \frac{x + \sin x}{1 + \cos x} dx$ |
| (B) | Find an equation of line through $(-4, 7)$ and parallel to the line $2x - 7y + 4 = 0$ |
| Q.7-(A) | Evaluate $\int_0^{\pi/4} \frac{\sec\theta}{\sin\theta + \cos\theta} d\theta$ |
| (B) | Graph the feasible region of the system of linear inequalities and find the corner points of $3x + 2y \geq 6$, $x + y \leq 4$, $x \geq 0$, $y \geq 0$ |
| Q.8-(A) | Find a joint equation of the straight lines through the origin perpendicular to the lines represented by $x^2 + xy - 6y^2 = 0$ |
| (B) | Find equation of the tangent drawn from $(0, 5)$ to $x^2 + y^2 = 16$ |
| Q.9-(A) | Find the centre, foci, eccentricity, vertices and equations of directrices of $\frac{y^2}{4} - x^2 = 1$ |
| (B) | Prove that : $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$ |